A BRIEF GUIDE TO "ABSOLUTE VALUE"

For High-School Students

This material is based on:

THINKING MATHEMATICS!
Volume 4: **Functions and Their Graphs**
Chapter 1 and *Chapter 3*

CONTENTS

ABSOLUTE VALUE AS DISTANCE

Let's start with a series of warm-up questions.

EXERCISE 1: An ant stands on a very long ruler. Every inch mark is marked with a whole number and the ant is currently standing at the position marked "8 inches."

a) How many inches is it from the ant to the 12 inch mark?

b) How many inches is it from the ant to the 4 inch mark?

c) How many inches is it from the ant to the -6 inch mark? (I said it was a very long ruler!)

d) How many inches it from the ant to the 243 inch mark?

e) How many inches is it between the 16 inch mark and the 89 inch mark?

f) How many inches are between the marks -12 and 30?

g) How many inches are between the marks 0 and 20?

h) How many inches are between the marks 0 and -20?

i) How many inches are between the marks -677 and 402? And how many inches are between the marks 402 and -677?

Is it possible to write a formula for the number of inches between the a inch mark and the b inch mark? What do you think? (Think carefully!)

EXERCISE 2:

a) What is the value of $-x$ if x is 16?

b) What is the value of $-x$ if x is -16?

c) What is the value of $-x$ if x is 0?

EXERCISE 3: The *radix* symbol $\sqrt{\ }$ is a symbol from geometry in which all quantities are considered positive: positive length, positive area, and so forth. (There are no negative quantities in geometry.) Thus, the radix refers only to the <u>positive</u> square root of a number. For example:

$$\sqrt{9} = 3 \qquad \sqrt{100} = 10 \qquad \sqrt{289} = 17$$

(Even though -3 is also a square root of 9, for instance, writing $\sqrt{9} = -3$ is technically incorrect. Even writing $\sqrt{9} = \pm 3$ is still technically incorrect!)

a) What is $\sqrt{169}$?

b) What is $\sqrt{400}$?

c) What is the square root of zero? Is zero considered positive or negative, both or neither? Could, or should, geometry allow quantities of measure zero?

d) Solve $x^2 = 169$. (Is there one or two solutions? Is this a geometry question or an algebra question?)

Now let's get a little strange!

e) What is the value of $\sqrt{7^2}$?

f) What is the value of $\sqrt{836^2}$?

g) What is the value of $\sqrt{(-7)^2}$?

h) What is the value of $\sqrt{(-78)^2}$?

i) What is the value of $\sqrt{x^2}$ if x is 95?

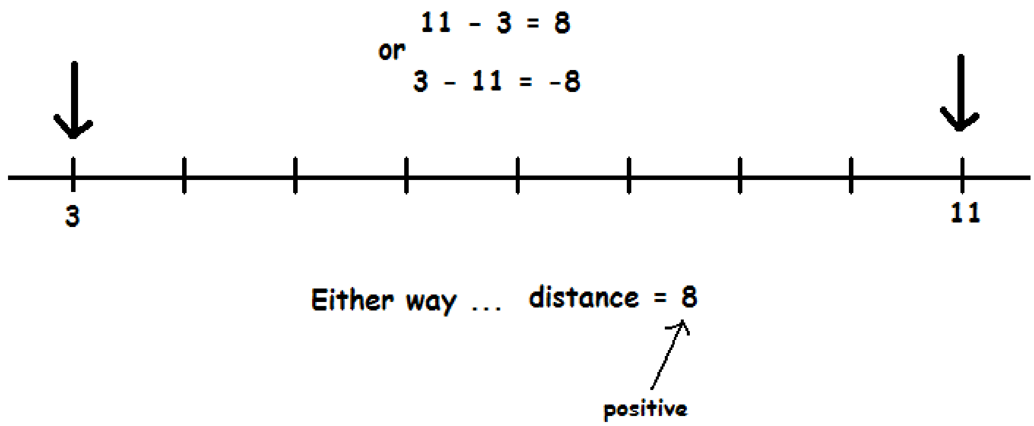
j) What is the value of $\sqrt{x^2}$ if x is -95?

k) What is the value of $\sqrt{x^2}$ if x is -38074?

l) What is the value of $\sqrt{x^2}$ if x is -0.0086?

In general, what can you say about the quantity $\sqrt{x^2}$ if x is a positive number? What can you say about the value of $\sqrt{x^2}$ if x is a negative number?

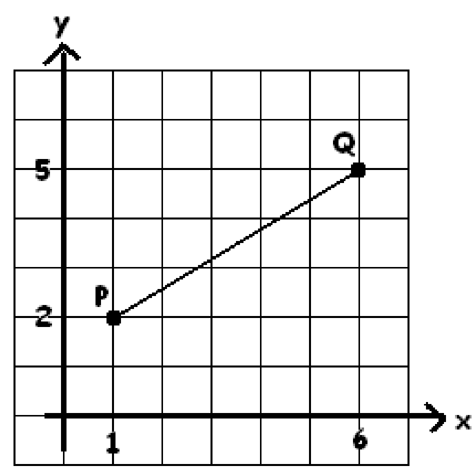
Exercise 1 shows that if we wish to compute the distance between any two points on a number line, we need to compute the positive difference of those two numbers.



Exercises 2 and 3 show that there are at least two ways to make sure answers are positive. We've already seen the second way in our geometry course. Recall …

THE DISTANCE FORMULA

Consider the points P and Q on the coordinate plane shown. What is the distance between them?



Pythagoras's theorem shows that the distance between points P and Q is:

$$d(P,Q)=\sqrt{5^2+3^2}=\sqrt{34}$$

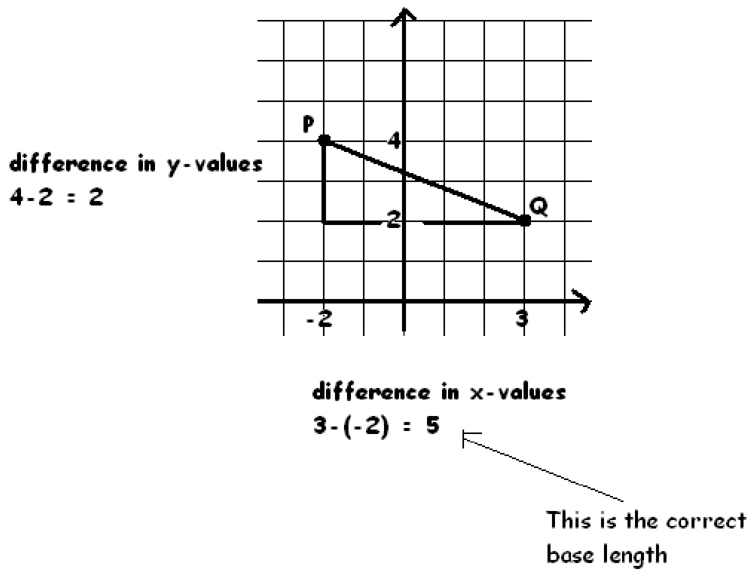
The radix here makes sure that the answer is positive – as it should be!

We have:

$$\text{distance } = \sqrt{\left(\text{difference in x-values}\right)^2 + \left(\text{difference in y-values}\right)^2}$$

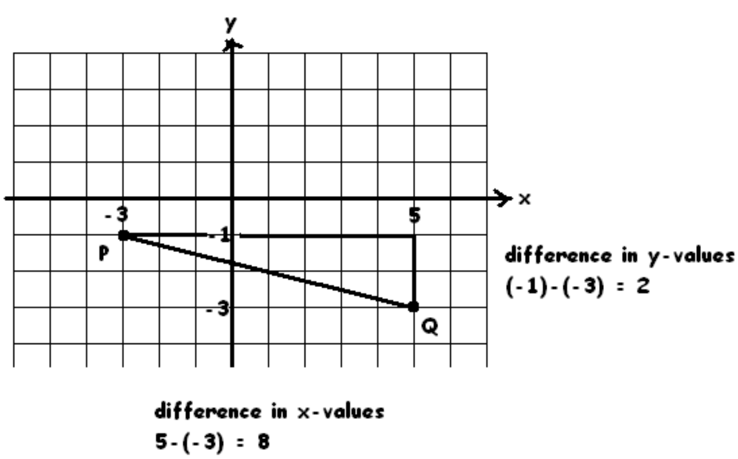
This is valid even if the points involved lie in alternative quadrants of the plane.

EXAMPLE:



$d(P,Q) = \sqrt{5^2 + 2^2} = \sqrt{29}$

EXAMPLE:



$d(P,Q) = \sqrt{8^2 + 2^2} = \sqrt{68}$