INTRODUCTION TO REAL ANALYSIS

William F. Trench
Andrew G. Cowles Distinguished Professor Emeritus
Department of Mathematics
Trinity University
San Antonio, Texas, USA
wtrench@trinity.edu

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FUNCTIONS DEFINED BY IMPROPER INTEGRALS

THE METHOD OF LAGRANGE MULTIPLIERS

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