

The Free High School Science Texts: A Textbook  
for High School Students Studying Maths.

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December 9, 2005

<sup>1</sup>See <http://savannah.nongnu.org/projects/fhsst>

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# Contents

<b>I</b>	<b>Maths</b>	<b>1</b>
<b>1</b>	<b>Numbers</b>	<b>3</b>
1.1	Letters and Arithmetic . . . . .	4
1.1.1	Adding and Subtracting . . . . .	4
1.1.2	Negative Numbers . . . . .	5
1.1.3	Brackets . . . . .	6
1.1.4	Multiplying and Dividing . . . . .	6
1.1.5	Rearranging Equations . . . . .	7
1.1.6	Living Without the Number Line . . . . .	8
1.2	Types of Real Numbers . . . . .	9
1.2.1	Integers . . . . .	9
1.2.2	Fractions and Decimal numbers . . . . .	9
1.2.3	Rational Numbers . . . . .	10
1.3	Exponents . . . . .	14
1.4	Surds . . . . .	16
1.4.1	Like and Unlike Surds . . . . .	17
1.4.2	Rationalising Denominators . . . . .	18
1.4.3	Estimating a Surd . . . . .	18
1.5	Accuracy . . . . .	19
1.5.1	Scientific Notation . . . . .	20
1.5.2	Worked Examples . . . . .	20
1.5.3	Exercises . . . . .	21
<b>2</b>	<b>Patterns in Numbers</b>	<b>22</b>
2.1	Sequences . . . . .	22
2.1.1	Arithmetic Sequences . . . . .	23
2.1.2	Quadratic Sequences . . . . .	25
2.1.3	Geometric Sequences . . . . .	26
2.1.4	Recursive Equations for sequences . . . . .	27
2.1.5	Extra . . . . .	28
2.2	Series ( <i>Grade 12</i> ) . . . . .	28
2.2.1	Finite Arithmetic Series . . . . .	29
2.2.2	Finite Squared Series . . . . .	32
2.2.3	Finite Geometric Series . . . . .	32
2.2.4	Infinite Series . . . . .	33
2.3	Worked Examples . . . . .	33
2.4	Exercises . . . . .	36

<b>3</b>	<b>Functions</b>	<b>38</b>
3.1	Functions and Graphs . . . . .	38
3.1.1	Variables, Constants and Relations . . . . .	39
3.1.2	Definition of a Function (grade 12) . . . . .	40
3.1.3	Domain and Range of a Relation . . . . .	41
3.1.4	Example Functions . . . . .	42
3.2	Exponentials and Logarithms . . . . .	46
3.2.1	Exponential Functions . . . . .	47
3.2.2	Logarithmic Functions . . . . .	47
3.3	Extra . . . . .	49
3.3.1	Absolute Value Functions . . . . .	49
<b>4</b>	<b>Numerics</b>	<b>53</b>
4.1	Optimisation . . . . .	53
4.1.1	Linear Programming . . . . .	55
4.2	Gradient . . . . .	57
4.3	Old Content (please delete when finished) . . . . .	57
4.3.1	Problems . . . . .	57
4.3.2	Maximising or Minimising the Objective Function . . . . .	59
	<b>Essay 1: Differentiation in the Financial World</b>	<b>62</b>
<b>5</b>	<b>Differentiation</b>	<b>64</b>
5.1	Limit and Derivative . . . . .	64
5.1.1	Gradients and limits . . . . .	64
5.1.2	Differentiating $f(x) = x^n$ . . . . .	66
5.1.3	Other notations . . . . .	67
5.2	Rules of Differentiation . . . . .	67
5.2.1	Summary . . . . .	68
5.3	Using Differentiation with Graphs . . . . .	68
5.3.1	Finding Tangent Lines . . . . .	68
5.3.2	Curve Sketching . . . . .	69
5.4	Worked Examples . . . . .	69
5.5	Exercises . . . . .	74
<b>6</b>	<b>Geometry</b>	<b>76</b>
6.1	Polygons . . . . .	76
6.1.1	Triangles . . . . .	77
6.1.2	Quadrilaterals . . . . .	78
6.1.3	Other polygons . . . . .	79
6.1.4	Similarity of Polygons . . . . .	79
6.1.5	Midpoint Theorem . . . . .	79
6.1.6	Extra . . . . .	80
6.2	Solids . . . . .	81
6.3	Coordinates . . . . .	82
6.4	Transformations . . . . .	82
6.4.1	Shifting, Reflecting, Stretching and Shrinking Graphs: . . . . .	82
6.5	Stretching and Shrinking Graphs . . . . .	84
6.6	Mixed Problems . . . . .	86
6.7	Equation of a Line . . . . .	86

6.8	Circles . . . . .	87
6.8.1	Circles & Semi-circles . . . . .	87
6.9	Locus . . . . .	88
6.10	Other Geometries . . . . .	88
6.11	Unsorted . . . . .	88
6.11.1	Fundamental vocabulary terms . . . . .	88
6.11.2	Parallel lines intersected by transversal lines . . . . .	91
<b>7</b>	<b>Trigonometry</b>	<b>96</b>
7.1	Syllabus . . . . .	96
7.1.1	Triangles . . . . .	96
7.1.2	Trigonometric Formulæ . . . . .	96
7.2	Radian and Degree Measure . . . . .	96
7.2.1	The unit of radians . . . . .	97
7.3	Definition of the Trigonometric Functions . . . . .	98
7.3.1	Trigonometry of a Right Angled Triangle . . . . .	98
7.3.2	Trigonometric Graphs . . . . .	100
7.3.3	Secant, Cosecant, Cotangent and their graphs . . . . .	102
7.3.4	Inverse trigonometric functions . . . . .	104
7.4	Trigonometric Rules and Identities . . . . .	105
7.4.1	Translation and Reflection . . . . .	105
7.4.2	Pythagorean Identities . . . . .	107
7.4.3	Sine Rule . . . . .	108
7.4.4	Cosine Rule . . . . .	111
7.4.5	Area Rule . . . . .	112
7.4.6	Addition and Subtraction Formulae . . . . .	113
7.4.7	Double and Triple Angle Formulae . . . . .	115
7.4.8	Half Angle Formulae . . . . .	116
7.4.9	‘Product to Sum’ and ‘Sum to Product’ Identities . . . . .	117
7.4.10	Solving Trigonometric Identities . . . . .	117
7.5	Application of Trigonometry . . . . .	118
7.5.1	Height and Depth . . . . .	118
7.5.2	Maps and Plans . . . . .	119
7.6	Trigonometric Equations . . . . .	120
7.6.1	Solution using CAST diagrams . . . . .	122
7.6.2	Solution Using Periodicity . . . . .	125
7.6.3	Linear Trigonometric Equations . . . . .	126
7.6.4	Quadratic and Higher Order Trigonometric Equations . . . . .	126
7.6.5	More Complex Trigonometric Equations . . . . .	127
7.7	Summary of the Trigonometric Rules and Identities . . . . .	128
<b>8</b>	<b>Solving Equations</b>	<b>130</b>
8.1	Linear Equations . . . . .	130
8.1.1	Introduction . . . . .	130
8.1.2	Solving Linear equations - the basics . . . . .	131
8.1.3	Solving linear equations - Combining the basics in a few steps . . . . .	133
8.2	Quadratic Equations . . . . .	136
8.2.1	The Quadratic Function . . . . .	136

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