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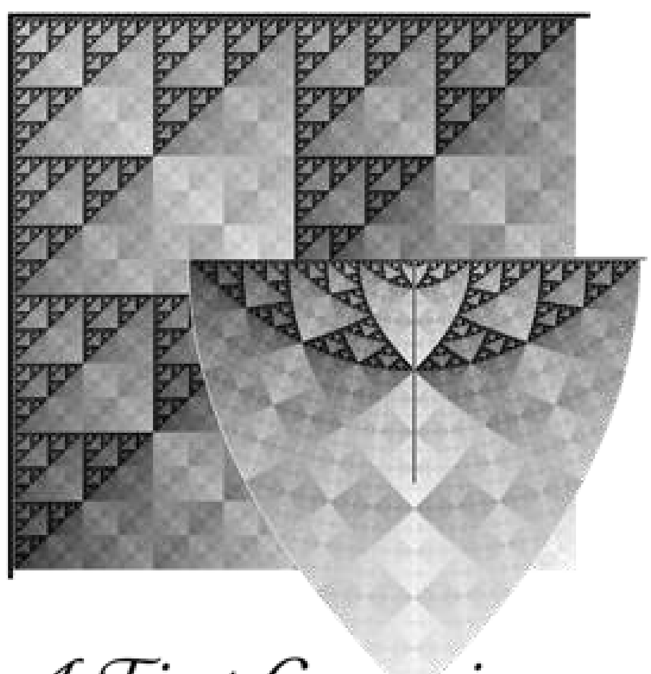

A First Course in COMPLEX ANALYSIS

Version 1.5

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The cover illustration, *Square Squared* by Robert Chaffer, shows two superimposed images. The foreground image represents the result of applying a transformation, $z \mapsto z^2$ (see Exercises 3.53 and 3.54), to the background image. The locally conformable property of this mapping can be observed through matching the line segments, angles, and Sierpinski triangle features of the background image with their respective images in the foreground figure. (The foreground figure is scaled down to about 40% and repositioned to accommodate artistic and visibility considerations.)

The background image fills the square with vertices at 0, 1, $1 + i$, and i (the positive direction along the imaginary axis is chosen as downward). It was prepared by using Michael Barnsley's chaos game, capitalizing on the fact that a square is self tiling, and by using a fractal-coloring method. (The original art piece is in color.) A subset of the image is seen as a standard Sierpinski triangle. The chaos game was also re-purposed to create the foreground image.

"And what is the use of a book," thought Alice, "without pictures or conversations?"
Lewis Carroll (*Alice in Wonderland*)

About this book. *A First Course in Complex Analysis* was written for a one-semester undergraduate course developed at Binghamton University (SUNY) and San Francisco State University, and has been adopted at several other institutions. For many of our students, Complex Analysis is their first rigorous analysis (if not mathematics) class they take, and this book reflects this very much. We tried to rely on as few concepts from real analysis as possible. In particular, series and sequences are treated from scratch, which has the consequence that power series are introduced late in the course. The goal our book works toward is the Residue Theorem, including some nontraditional applications from both continuous and discrete mathematics.

A printed paperback version of this open textbook is available from Orthogonal Publishing (`www.orthogonalpublishing.com`) or your favorite online bookseller.

About the authors. Matthias Beck is a professor in the Mathematics Department at San Francisco State University. His research interests are in geometric combinatorics and analytic number theory. He is the author of two other books, *Computing the Continuous Discretely: Ingeger-point Enumeration in Polyhedra* (with Sinai Robins, Springer 2007) and *The Art of Proof: Basic Training for Deeper Mathematics* (with Ross Geoghegan, Springer 2010).

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Robert Chaffer (cover art) is a professor emeritus at Central Michigan University. His academic interests are in abstract algebra, combinatorics, geometry, and computer applications. Since retirement from teaching, he has devoted much of his time to applying those interests to creation of art images (`people.cst.cmich.edu/chaff1ra/Art_From_Mathematics`).

A Note to Instructors. The material in this book should be more than enough for a typical semester-long undergraduate course in complex analysis; our experience taught us that there is more content in this book than fits into one semester. Depending on the nature of your course and its place in your department's overall curriculum, some sections can be either partially omitted or their definitions and theorems can be assumed true without delving into proofs. Chapter 10 contains optional longer homework problems that could also be used as group projects at the end of a course.

We would be happy to hear from anyone who has adopted our book for their course, as well as suggestions, corrections, or other comments.

Acknowledgements. We thank our students who made many suggestions for and found errors in the text. Special thanks go to Collin Bleak, Matthew Brin, John McCleary, Sharma Pallekonda, Joshua Palmatier, and Dmytro Savchuk for comments, suggestions, and additions after teaching from this book.

We thank Lon Mitchell for his initiative and support for the print version of our book with Orthogonal Publishing, and Bob Chaffer for allowing us to feature his art on the book's cover.

We are grateful to the American Institute of Mathematics for including our book in their Open Textbook Initiative (`aimath.org/textbooks`).

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