A First Course in Linear Algebra

Robert A. Beezer

University of Puget Sound

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Robert A. Beezer is a Professor of Mathematics at the University of Puget Sound, where he has been on the faculty since 1984. He received a B.S. in Mathematics (with an Emphasis in Computer Science) from the University of Santa Clara in 1978, a M.S. in Statistics from the University of Illinois at Urbana-Champaign in 1982 and a Ph.D. in Mathematics from the University of Illinois at Urbana-Champaign in 1984.

In addition to his teaching at the University of Puget Sound, he has made sabbatical visits to the University of the West Indies (Trinidad campus) and the University of Western Australia. He has also given several courses in the Master's program at the African Institute for Mathematical Sciences, South Africa. He has been a Sage developer since 2008.

He teaches calculus, linear algebra and abstract algebra regularly, while his research interests include the applications of linear algebra to graph theory. His professional website is at http://buzzard.ups.edu.

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The most recent version can always be found at http://linear.pugetsound.edu.

To my wife, Pat.

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