CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Steps to Solving Equations

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley
Beta Version

Steps to Solving Equations

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Form and solve linear equations involving factorizing and using the distributive law.

In particular, this unit aims to help you identify and assist students who have difficulties in:

- Using variables to represent quantities in a real-world or mathematical problem.
- Solving word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

7.EE: Use properties of operations to generate equivalent expressions.

Solve real-life and mathematical problems using numerical and algebraic expressions and equations

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
4. Model with mathematics.

INTRODUCTION

The lesson unit is structured in the following way:

- Before the lesson, students attempt the assessment task individually. You then review students' responses and formulate questions that will help them improve their work.
- During the lesson, students work collaboratively in pairs or threes, matching equations to stories and then ordering the steps used to solve these equations. Throughout their work, students explain their reasoning to their peers.
- Finally, students again work individually to review their work and attempt a second task, similar to the initial assessment task.

MATERIALS REQUIRED

- Each student will need copies of the assessment tasks *Express Yourself* and *Express Yourself (revisited)*, and *Card Set: Stories* (not cut up), a mini-whiteboard, a pen, and an eraser.
- For each small group of students provide cut up copies of *Card Set: Stories* (cut up), *Card Set: Equations*, and *Card Set: Steps to Solving*, a large sheet of paper for making a poster, a marker, and a glue stick.
- There are also some projector resources to help with whole-class discussion.

TIME NEEDED

15 minutes before the lesson for the assessment task, a 1-hour lesson, and 15 minutes in a follow-up lesson (or for homework). All timings are approximate, depending on the needs of your students.

BEFORE THE LESSON

Assessment task: *Express Yourself* (15 minutes)

Have the students do this task, in class or for homework, a day or more before the formative assessment lesson. This will give you an opportunity to assess the work, and identify students who have misconceptions or need other forms of help. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of *Express Yourself*. Introduce the task briefly and help the class to understand what they are being asked to do.

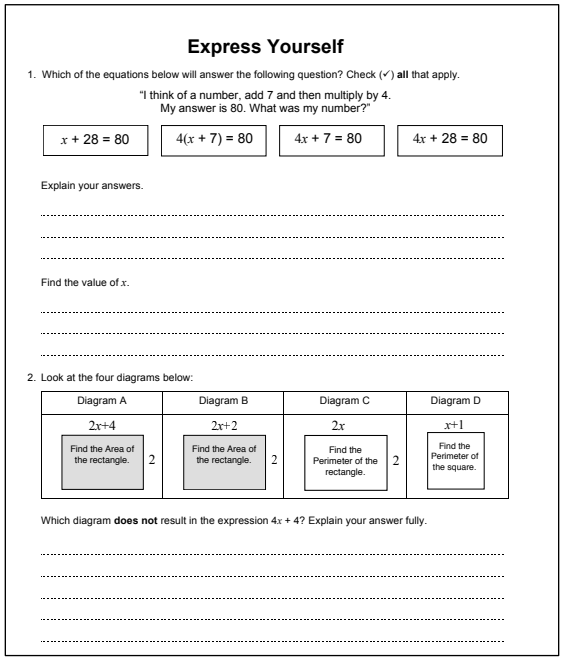
Spend 15 minutes working individually, answering these questions.

Show all your work on the sheet.

Make sure you explain your answers really clearly.

It is important that, as far as possible, students answer the questions without assistance.

Students should not worry too much if they cannot understand or do everything because you will teach a lesson using a similar task, which should help them. Explain to students that, by the end of the next lesson, they should expect to answer questions such as these. This is their goal.



Express Yourself

1. Which of the equations below will answer the following question? Check (✓) **all** that apply.

"I think of a number, add 7 and then multiply by 4. My answer is 80. What was my number?"

$x + 28 = 80$	$4(x + 7) = 80$	$4x + 7 = 80$	$4x + 28 = 80$

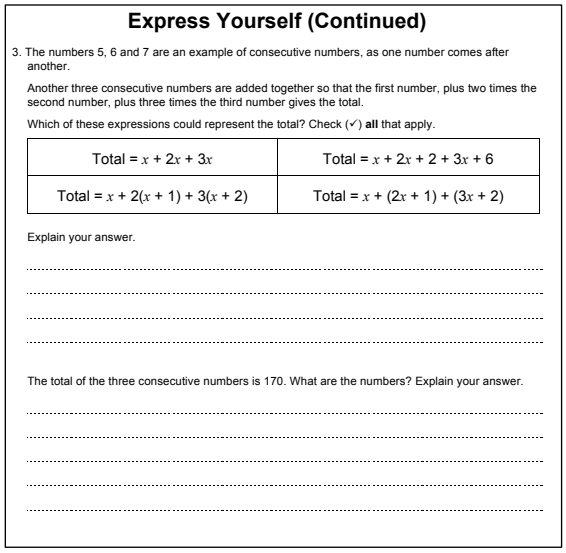
Explain your answers.

Find the value of x.

2. Look at the four diagrams below:

Diagram A	Diagram B	Diagram C	Diagram D
$2x+4$	$2x+2$	$2x$	$x+1$
Find the Area of the rectangle. 2	Find the Area of the rectangle. 2	Find the Perimeter of the rectangle. 2	Find the Perimeter of the square.

Which diagram **does not** result in the expression $4x + 4$? Explain your answer fully.



Express Yourself (Continued)

3. The numbers 5, 6 and 7 are an example of consecutive numbers, as one number comes after another.

Another three consecutive numbers are added together so that the first number, plus two times the second number, plus three times the third number gives the total.

Which of these expressions could represent the total? Check (✓) **all** that apply.

Total = $x + 2x + 3x$	Total = $x + 2x + 2 + 3x + 6$
Total = $x + 2(x + 1) + 3(x + 2)$	Total = $x + (2x + 1) + (3x + 2)$

Explain your answer.

The total of the three consecutive numbers is 170. What are the numbers? Explain your answer.

Assessing students' responses

Collect students' responses to the task. Make some notes on what their work reveals about their current levels of understanding and difficulties. The purpose of this is to forewarn you of the issues that will arise during the lesson, so that you may prepare carefully.

We suggest that you do not score students' work. Research shows that this is counterproductive as it encourages students to compare scores, and distracts their attention from how they may improve their mathematics.

Instead, help students to make further progress by asking questions that focus attention on aspects of their work. Some suggestions for these are given in the table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We strongly recommend that you write your own lists of questions, based on your students' work, using the ideas in the *Common issues* table. You may choose to write questions on each student's work. If you do not have time for this, then prepare a few questions that apply to most students and write these on the board when the assessment task is revisited.

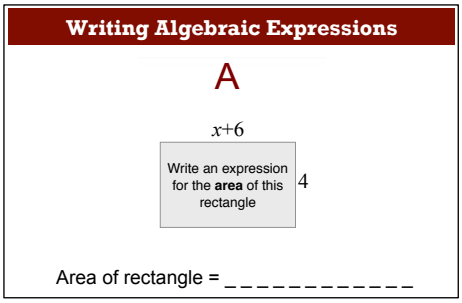
Common issues:	Suggested questions and prompts:
Student applies operations in the wrong order (Q1) For example: The student chooses $4x + 7 = 80$ as an appropriate equation. Or: The student chooses $x + 28 = 80$ as an appropriate equation.	• In this expression, what is the first thing that happens to the number I am thinking of? Then what happens? • What does x represent? What are you adding 7 to? • Is adding 7 and then multiplying by 4 the same as adding 28? How could you check this?
Student does not recognize all relevant expressions (Q1) For example: The student chooses $4(x + 7) = 80$ as the only appropriate equation.	• How else could you write the expression $4(x + 7)$?
Student does not distinguish between area and perimeter (Q2) For example: The student writes an expression for the area instead of the perimeter of the rectangles in Diagrams C and D. Or: The student writes an expression for the perimeter instead of the area of the rectangles in Diagrams A and B.	• How do you calculate the area of a rectangle? • What does perimeter mean? • Does your expression represent the area or the perimeter of this rectangle?
Student assumes the three numbers are equal (Q3) For example: The student selects Total = $x + 2x + 3x$ as an appropriate expression.	• What does 'consecutive' mean? • What does x represent? • Can you try some numbers to check that this works?
Student does not multiply all terms in the bracket (Q3) For example: The student selects Total = $x + (2x + 1) + (3x + 2)$ as an appropriate expression.	• What does x represent? • How do you write 'one more than x' using algebra? Now read the question again: what happens next? What happens if you add two of these numbers together?
Student calculates an incorrect value for x (Q1, Q3)	• If you substitute your value of x into the left hand side of the equation, does it equal the number on the right hand side? • How will you check whether your value for x is correct?
Student does not interpret the solution For example: The student does not realize that x represents the number first thought of (Q1). Or: The student does not recognize that $x = 27$ is the first of the three consecutive numbers found (Q3).	• You have found that $x = 27$. Read the question again. What are the **three** consecutive numbers?
Student completes the task	• Can you make up a situation that would lead to the equation $4(x + 3) = 16$? • Could you solve these equations using a different method? What would the method be?

SUGGESTED LESSON OUTLINE

Whole-class introduction: (10 minutes)

Give each student a mini-whiteboard, pen, and an eraser.

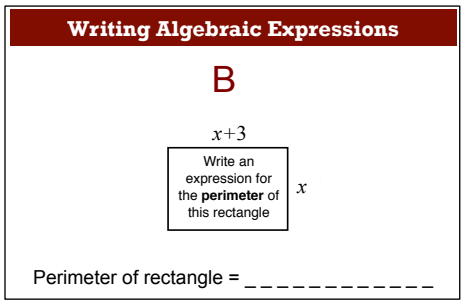
Display Slide P-1 of the projector resource.



Write an expression for the area of this rectangle on your whiteboard.

Spend time discussing the expressions students give. Some students may write the expression $4(x + 6)$ whereas others may apply the distributive law to give $4x + 24$. Explain their equivalence by considering how the area of the single rectangle $4(x+6)$ may be split into the two smaller areas $4x$ and 24 by drawing a vertical line. Notice whether students make the mistake of writing the expression as $4x + 6$ or whether they confuse the area of the rectangle with the perimeter.

Display Slide P-2 of the projector resource:



Write an expression for the perimeter of this rectangle on your whiteboard.

Again, spend time discussing the expressions given by students. Notice whether students collect like terms to give $2(2x + 3)$ or $4x + 6$, or whether they give an un-simplified expression, for example, $x + 3 + x + x + 3 + x$. Display Slide P-3 of the projector resource: