CONCEPT DEVELOPMENT

Mathematics Assessment Project

CLASSROOM CHALLENGES

A Formative Assessment Lesson

Solving Linear Equations in One Variable

Mathematics Assessment Resource Service
University of Nottingham & UC Berkeley

Beta Version

Solving Linear Equations in One Variable

MATHEMATICAL GOALS

This lesson unit is intended to help you assess how well students are able to:

- Solve linear equations in one variable with rational number coefficients.
- Collect like terms.
- Expand expressions using the distributive property.
- Categorize linear equations in one variable as having one, none, or infinitely many solutions.

It also aims to encourage discussion on some common misconceptions about algebra.

COMMON CORE STATE STANDARDS

This lesson relates to the following *Standards for Mathematical Content* in the *Common Core State Standards for Mathematics*:

8.EE: Expressions and Equations.

This lesson also relates to the following *Standards for Mathematical Practice* in the *Common Core State Standards for Mathematics*:

1. Make sense of problems and persevere in solving them.
3. Construct viable arguments and critique the reasoning of others.
7. Look for and make use of structure.

INTRODUCTION

This lesson unit is structured in the following way:

- Before the lesson, students work individually on an assessment task that is designed to reveal their current understanding and difficulties. You then review their responses and create questions for students to consider when improving their work.s
- After a whole-class introduction, students work in small groups on a collaborative discussion task, categorizing equations based on the number of solutions. Throughout their work, students justify and explain their thinking and reasoning.
- In the same small groups, students critique the work of others and then discuss as a whole class what they have learned.
- Finally, students return to their original task and try to improve their own, individual responses.

MATERIALS REQUIRED

- Each student will need two copies of the assessment task *When are the equations true?,* a mini-whiteboard, a pen, and an eraser and a copy of *When are the equations true? (revisited)*.
- Each small group of students will need *Card Set*: *Equations*, a pair of scissors, a pencil, a marker, a glue stick, and a large sheet of paper for making a poster.
- There is a projector resource to support the whole-class introduction.

TIME NEEDED

15 minutes before the lesson for the assessment task, a 70-minute lesson, and 10 minutes in a follow-up lesson (or for homework). Timings are approximate. Exact timings will depend on the needs of the class.

BEFORE THE LESSON

Assessment task: *When are the equations true?* (15 minutes)

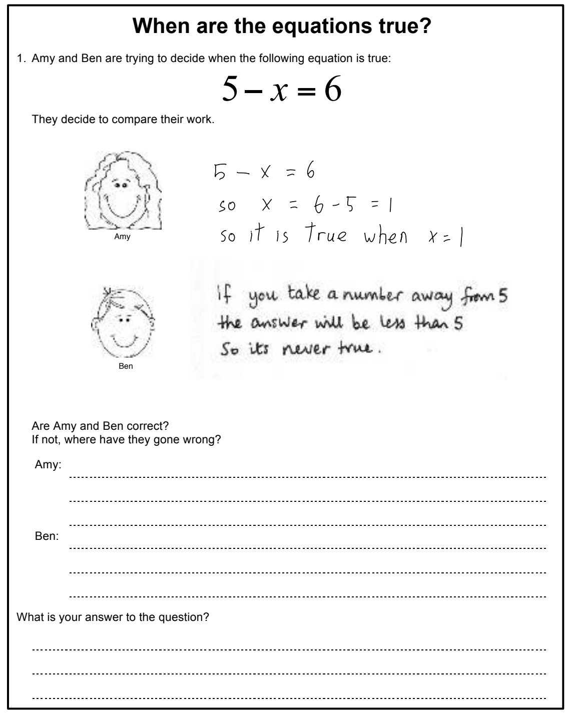
Ask students complete this task, in class or for homework, a few days before the formative assessment lesson. This will give you an opportunity to assess the work and to find out the kinds of difficulties students have with it. You should then be able to target your help more effectively in the follow-up lesson.

Give each student a copy of the assessment task.

> *Spend 15 minutes working individually, answering these questions. Make sure you explain your answers really clearly.*

It is important that, as far as possible, students are allowed to answer the questions without assistance.

Students should not worry too much if they cannot understand or do everything because, in the next lesson, they will work on a similar task that should help them. Explain to students that, by the end of the next lesson, they should be able to answer questions such as these confidently. This is their goal.



When are the equations true?

1. Amy and Ben are trying to decide when the following equation is true:

$$5 - x = 6$$

They decide to compare their work.

Amy:

$5 - x = 6$
so $x = 6 - 5 = 1$
so it is true when $x = 1$

Ben:

If you take a number away from 5
the answer will be less than 5
So its never true.

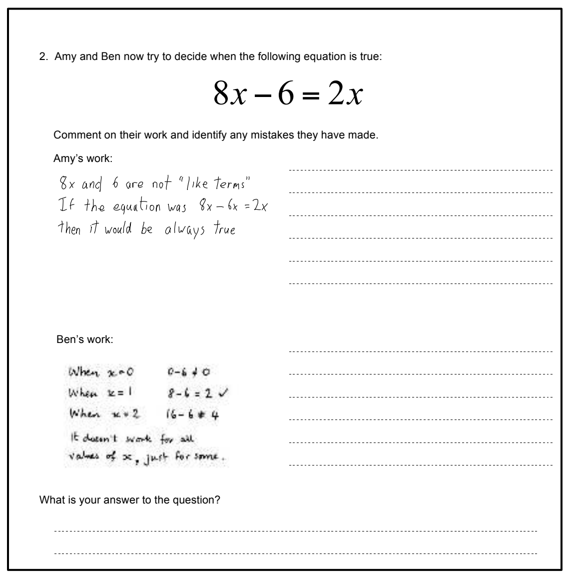
Are Amy and Ben correct?
If not, where have they gone wrong?

Amy: ......

Ben: ......

What is your answer to the question?

......



2. Amy and Ben now try to decide when the following equation is true:

$$8x - 6 = 2x$$

Comment on their work and identify any mistakes they have made.

Amy's work:

8x and 6 are not "like terms"
If the equation was $8x - 6x = 2x$
then it would be always true

Ben's work:

When $x = 0$ $0 - 6 \neq 0$
When $x = 1$ $8 - 6 = 2$ ✓
When $x = 2$ $16 - 6 \neq 4$
It doesn't work for all values of x, just for some.

What is your answer to the question?

......

Assessing students' responses

Collect students' responses to the task, and make some notes on what their work reveals about their current levels of understanding. The purpose of doing this is to forewarn you of the difficulties students will experience during the lesson itself, so that you may prepare carefully.

We suggest that you do not write scores on students' work. Research shows that this is counterproductive, as it encourages students to compare scores, and distracts their attention from what they can do to improve their mathematics.

Instead, help students to make further progress by asking questions that focus their attention on aspects of their work. Some suggestions for these are given in the *Common issues* table on the next page. These have been drawn from common difficulties observed in trials of this unit.

We suggest you make a list of your own questions, based on your students' work. We recommend you either:

- Write one or two questions on each student's work, or
- Give each student a printed version of your list of questions, and highlight appropriate questions for each student.

If you do not have time to do this, you could select a few questions that will be of help to the majority of students, and write these on the board when you return the work to the students.

Common issues:	**Suggested questions and prompts:**
Student assumes that subtraction is commutative For example: Assumes that $5 - x = 6$ is the same as $x - 5 = 6$ and gives a value of $x = 11$ (Q1).	• Is 3 – 2 the same as 2 – 3? Try some other numbers. Will it ever be the same? • Now look at your work. Is $5 - x$ the same as $x - 5$? How do you know? • Check your work by substituting $x = 11$ back into the equation. What do you notice?
Student considers positive numbers only For example: Assumes that Ben is correct (Q1).	• Consider this: 3 subtract 2 equals 1; 3 subtract 1 equals 2; 3 subtract 0 equals 3. Can you see any patterns? What comes next? Three subtract something equals four?
Student fails to explain why Amy and Ben are incorrect (Q1)	• How can you show that Amy's value of x does not satisfy the equation? • How could you convince someone else that Amy/Ben has made a mistake? How could you show this?
Student understands both Amy and Ben are incorrect, but does not provide a correct solution	• What math can you use to work out if there is a value for x that satisfies the equation? [Guess and check or make x the subject of the equation.]
Incorrect use of the equal sign For example: The student writes $5 - x = 6 = x = -1$.	• Carefully check your work. In your head say the math you've written. Does it make sense if you read it from left to right?
Student solves the equation to give $x = -1$ correctly and states the equation is true	• Is this equation always true? • How many solutions does this equation have? • How do you know?
Student assumes Amy is correct (Q2)	• Give x a value. Are the terms alike now? Can you now subtract 6 from $8x$? • Can you find a value for x that makes the equation true?
Student assumes Ben is correct (Q2)	• How many solutions does an equation need to have, to be true? • Is $x = 1$ the only solution to this equation? How do you know?
Student completes the task	• Can you think of a different way of showing when the equation is true? Which method is the most convincing? Why do you think this is? • Can you write down a new equation, that is always/never true? How could you prove this? • Can you think of an equation that is sometimes true, but has more than one solution?

SUGGESTED LESSON OUTLINE

Whole-class introduction (15 minutes)

Give each student a mini-whiteboard, a pen, and an eraser. Maximize participation in the discussion by asking all students to show you their solutions on their mini-whiteboards.

This introduction will provide students with a model of how they should justify their solutions in the collaborative activity.

Display Slide P-1 of the projector resource:

> **True or False?**
>
> $$4x + 1 = 3$$
>
> Can you give me a value for x that makes this equation **false**?
>
> Show the calculations that explain your answer.

Students should not have any problems with finding a suitable value for x, but may not be too adventurous in their choices. Spend some time discussing the values given and the reasons for each choice, identifying any common choices, as well as any calculation errors.

Display Slide P-2 of the projector resource:

> **True or False?**
>
> $$4x + 1 = 3$$
>
> Can you give me a value for x that makes this equation **true**?
>
> Show the calculations that explain your answer.

Students may struggle with this at first, especially if their chosen method is substituting values for x. Encourage students to explore fractions, decimals and negative numbers as well as positive whole numbers.

If students find a value for x, challenge them to consider if there are any other values of x. They should be encouraged to justify why this is the only value for x that makes the equation true and how they can be sure of this.

Would we describe this equation as always true, never true or sometimes true? [Sometimes true.]

When is it true? [When $x = ½$.]

Are there any other values for x that make the equation true? How do you know?