| Grade Level/Course: Algebra 1 |
|---|
| Lesson/Unit Plan Name: Solving Radical Equations |
| Rationale/Lesson Abstract: |
| To provide students the best possible methods for solving equations that contains a radical. |
| Timeframe: 60 minutes for solving an equation with only one radical, 100 minutes for solving an equation with more than one radical. |
| Common Core Standard(s): |
| A-REI-2: Solve simple rational and radical equations in one variable and give examples how extraneous solutions may arise. |
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Instructional Resources/Materials:

One set of activity cards per pair of students. See activity at the end of lesson.

Lesson:

"These are some examples and non-examples of radical equations. Talk with your elbow partner and come up with a sentence that defines a radical equation." [Have students share out before giving formal definition]

| Examples of Radical Equations | Non-Examples of Radical Equations |
|-------------------------------|-----------------------------------|
| $\sqrt{x} + 5 = 11$ | $\sqrt{5} + x^2 = 11$ |
| $\sqrt[3]{x-4} = 7$ | $x - 4 = \sqrt[4]{16}$ |
| $4\sqrt{x-7} + 12 = 28$ | $x\sqrt{10-7} + 12 = 28$ |
| $\sqrt[5]{x} = 225$ | $x^4 = \sqrt[3]{27}$ |

Definition of a Radical Equation: An equation where the variable is found underneath a square root, cube root or a higher root.

Definition of a Radicand: The number or expression under a radical symbol.

Ask students:

"What do we know about solving equations?" [Isolate the variable]

"Right, we do need to isolate the variable. What are some methods for isolating the variable?" [Decompose, inverse operations, bar model].

"Let's look at an equation with a variable as a radicand."

$$\sqrt{x} = 6$$

"For what value of x would you substitute in to make this equation a true statement?" [36]

"36, right, the $\sqrt{36} = 6$. Let's see all the algebra we could use to get to that answer."

$$\sqrt{x} = 6$$

$$\sqrt{x} = \sqrt{36}$$

$$x = 36$$

$$\sqrt{x} = 6$$

$$(\sqrt{x})^2 = 6^2$$

$$x = 36$$

$$(x^{\frac{1}{2}})^2 = (6)^2$$

$$x = 36$$

"Let's solve some equations!"

| Ex. 1 $\sqrt{x} + 5 = 11$ | | |
|----------------------------------|--|---|
| $\sqrt{x} + 5 = 11$ | $\sqrt{x} + 5 = 11$ | $\sqrt{x} + 5 = 11$ |
| $\sqrt{x} + 5 = 6 + 5$ | $\sqrt{x} + 5 - 5 = 11 - 5$ | $\sqrt{x} + 5 - 5 = 11 - 5$ |
| $\sqrt{x} + 5 = 6 + 5$ | $\sqrt{x} = 6$ | $\sqrt{x} = 6$ |
| $\sqrt{x} = 6$ | $\left(\sqrt{x}\right)^2 = \left(6\right)^2$ | $x^{\frac{1}{2}} = 6$ |
| $\sqrt{x} = \sqrt{36}$ $x = 36$ | x = 36 | $\left(x^{\frac{1}{2}}\right)^2 = \left(6\right)^2$ |
| | Check! | |
| Check! | | <i>x</i> = 36 |
| | | Check! |

| You Try: $\sqrt{x} - 4 = 7$ | | |
|------------------------------------|---|--|
| $\sqrt{x} - 4 = 7$ | $\sqrt{x} - 4 = 7$ | $\sqrt{x} - 4 = 7$ |
| $\sqrt{x} - 4 = 7 + 4 - 4$ | $\sqrt{x} - 4 + 4 = 7 + 4$ | $\sqrt{x} - 4 + 4 = 7 + 4$ |
| $\sqrt{x} - 4 = 7 + 4 - 4$ | $\sqrt{x} = 11$ | $\sqrt{x} = 11$ |
| $\sqrt{x} = 11$ | $\left(\sqrt{x}\right)^2 = \left(11\right)^2$ | $x^{\frac{1}{2}} = 11$ |
| $\sqrt{x} = \sqrt{121}$ | x = 121 | $\left(\begin{array}{c} 1 \\ 2 \end{array}\right)^2$ |
| x = 121 | Check! | $\left(x^{\frac{1}{2}}\right)^2 = \left(11\right)^2$ |
| Check! | Check: | x = 121 |
| | | Check! |

[&]quot;What is the difference between $\sqrt{x}-4=7$ and $\sqrt{x-4}=7$? "[The first has x as the radicand and -4 outside the square root and the other has the quantity x-4 as the radicand.]

"Let's solve $\sqrt{x-4} = 7$ and compare the answer to $\sqrt{x-4} = 7$ "

| Ex 2: $\sqrt{x-4} = 7$ | | |
|-------------------------------------|--|--|
| $\sqrt{x-4} = 7$ | $\sqrt{x-4} = 7$ | $\sqrt{x-4} = 7$ |
| $\sqrt{x-4} = \sqrt{49}$ $x-4 = 49$ | $\left(\sqrt{x-4}\right)^2 = \left(7\right)^2$ | $\left[\left(x - 4 \right)^{\frac{1}{2}} \right]^2 = \left(7 \right)^2$ |
| x-4=49+4-4 | x - 4 = 49 $x - 4 + 4 = 49 + 4$ | $ \begin{array}{c} 1 \\ x - 4 = 49 \end{array} $ |
| x - 4 = 49 + 4 - 4 $x = 53$ | x = 53 Check! | x-4+4=49+4 $x=53$ |
| Check! | | Check! |

| You Try: $\sqrt{x+6} = 15$ | | |
|---|--|---|
| $\sqrt{x+6} = 15$ | $\sqrt{x+6} = 15$ | $\sqrt{x+6} = 15$ |
| $\sqrt{x+6} = \sqrt{225}$ $x+6 = 225$ $x+6 = 219+6$ $x+6 = 219+6$ $x = 219$ | $(\sqrt{x+6})^2 = (15)^2$ $x+6 = 225$ $x+6-6 = 225-6$ $x = 219$ Check! | $\left[(x+6)^{\frac{1}{2}} \right]^2 = (15)^2$ $x+6 = 225$ $x+6-6 = 225-6$ $x = 219$ |
| Check! | | Check! |

Ex 3: $4\sqrt{x-7} + 12 = 28$ "What terms make up the radicand?" [x-7] "So, what do we need to isolate?" $[\sqrt{x-7}]$

$$4\sqrt{x-7} + 12 = 28$$

$$4\sqrt{x-7} + 12 = 16 + 12$$

$$4\sqrt{x-7} + 1/2 = 16 + 1/2$$

$$4\sqrt{x-7} = 16$$

$$4 \cdot \sqrt{x-7} = 4 \cdot 4$$

$$4 \cdot$$

Check!

$$4\sqrt{x-7} + 12 = 28$$

$$4\sqrt{x-7} + 12 - 12 = 28 - 12$$

$$4\sqrt{x-7} = 16$$

$$4\sqrt{x-7} = \frac{16}{4}$$

$$\sqrt{x-7} = 4$$

$$(\sqrt{x-7})^2 = (4)^2$$

$$x-7 = 16$$

$$x-7+7=16+7$$

$$x = 23$$

Check!

$$4\sqrt{x-7} + 12 = 28$$

$$\frac{4\sqrt{x-7}}{4} + \frac{12}{4} = \frac{28}{4}$$

$$\sqrt{x-7} + 3 = 7$$

$$\sqrt{x-7} + 3 - 3 = 7 - 3$$

$$\sqrt{x-7} = 4$$

$$(x-7)^{\frac{1}{2}} = 4$$

$$((x-7)^{\frac{1}{2}})^2 = 4^2$$

$$x-7 = 16$$

$$x-7+7=16+7$$

$$x = 23$$

Check!

| You Try: (use any method) $5\sqrt{x+3} - 10 = 15$ | | |
|--|--|--|
| $5\sqrt{x+3} - 10 = 15$ | $5\sqrt{x+3} - 10 = 15$ | $5\sqrt{x+3} - 10 = 15$ |
| $5\sqrt{x+3} - 10 = 15 - 10 + 10$ | $5\sqrt{x+3} - 10 + 10 = 15 + 10$ | $\frac{5\sqrt{x+3}}{2} - \frac{10}{2} = \frac{15}{2}$ |
| $5\sqrt{x+3} - 10 = 15 - 10 + 10$ | $5\sqrt{x+3} = 25$ | $\frac{-}{5} = \frac{-}{5} = \frac{-}{5}$ |
| $5\sqrt{x+3} = 25$ | $5\sqrt{x+3} \ \ 25$ | $\sqrt{x+3-2}=3$ |
| $5 \cdot \sqrt{x+3} = 5 \cdot 5$ | 5 = 5 | $\sqrt{x+3} - 2 + 2 = 3 + 2$ |
| $\cancel{5} \cdot \sqrt{x+3} = 5\cancel{5}$ | $\sqrt{x+3} = 5$ | $\sqrt{x+3} = 5$ |
| $\sqrt{x+3} = 5$ | $\left(\sqrt{x+3}\right)^2 = \left(5\right)^2$ | $\left(x+3\right)^{\frac{1}{2}}=5$ |
| $\sqrt{x+3} = \sqrt{25}$ | x + 3 = 25 | $\left(\left(x+3\right)^{\frac{1}{2}}\right)^2 = \left(5\right)^2$ |
| x + 3 = 25 | x+3-3=25-3 | $\left(\begin{pmatrix} x+3 \end{pmatrix}\right) = \begin{pmatrix} 3 \end{pmatrix}$ |
| x+3=22+3 | <i>x</i> = 22 | x + 3 = 25 |
| x + 3 = 22 + 3 $x = 22$ | Check! | x + 3 - 3 = 25 - 3 |
| x - 22 | | <i>x</i> = 22 |
| Check! | | Check! |

"What if our equation was $\sqrt{x} = -5$? Can you think of a number that when you take the square root gives you -5?" [25? No, -25! There isn't one]

"Right, there isn't a number in the Real Number Set that when you take the square root gives you –5 as an answer. In this case we would write 'no real solution'."

"With your partner, describe and correct the error you see in this problem. Check the answer shown to justify your response."

$$\sqrt{3x} + 9 = 0$$

$$\sqrt{3x} + 9 - 9 = 0 - 9$$

$$\sqrt{3x} = -9$$

$$\left(\sqrt{3x}\right)^2 = \left(-9\right)^2$$

$$3x = 81$$

$$3 \cdot x = 3 \cdot 27$$

$$x = 27$$

[The problem is $\sqrt{3x} = -9$. At this point there is no solution in the real numbers.]

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