

**MATHEMATICAL ANALYSIS I
(DIFFERENTIAL CALCULUS)
FOR ENGINEERS AND
BEGINNING
MATHEMATICIANS**

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To my family.

To those unknown people who by hard and honest working make possible our daily life of thinking.

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Preface

I start this preface with some ideas of my former Teacher and Master, senior researcher I, corresponding member of the Romanian Academy, Dr. Doc. Nicolae Popescu (Institute of Mathematics of the Romanian Academy).

Question: What is Mathematics?

Answer: It is the art of reasoning, thinking or making judgements. It is difficult to say more, because we are not able to exactly define the notion of a "table", not to say Math! In the greek language "mathema" means "knowledge". Do you think that there is somebody who is able to define this last notion? And so on... Let us do Math, let us apply or teach it and let us stop to search for a definition of it!

Q: Is Math like Music?

A: Since any human activity involves more or less need of reasoning, Mathematics is more connected with our everyday life then all the other arts. Moreover, any description of the natural or social phenomena use mathematical tools.

Q: What kind of Mathematics is useful for an engineer?

A: Firstly, the basic Analysis, because this one is the best tool for strengthening the ability of making correct judgements and of taking appropriate decisions. Formulas and notions of Analysis are at the basis of the particular language used by the engineering topics like Mechanics, Material Sciences, Elasticity, Concrete Sciences, etc. Secondly, Linear Algebra and Geometry develop the ability to work with vectors, with geometrical object, to understand some specific algebraic structures and to use them for applying some numerical methods. Differential Equations, Calculus of Variations and Probability Theory have a direct impact in the scientific presentation of all the engineering applications. Computer Science cannot be taught without the basic knowledge of the above mathematical topics. Mathematics comes from reality and returns to it.

Q: How can we learn Math such that this one not becomes abstract, annoying, difficult, etc.?

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