

MATHEMATICAL ANALYSIS II
INTEGRAL CALCULUS

SEVER ANGEL POPESCU

To my family.

.....

.....

To those who really try to put their lives in the Light of
Truth.

.....
Dedicated to the memory of my late Professor and
Mentor, Dr. Doc. Nicolae Popescu.

Contents

Preface	7
Chapter 1. Indefinite integrals (Primitives, Antiderivatives)	1
1. Definitions, some properties and basic formulas	1
2. Some results on polynomials	10
3. Primitives of rational functions	21
4. Primitives of irrational and trigonometric functions	27
5. Problems and exercises	47
Chapter 2. Definite integrals	49
1. The mass of a linear bar	49
2. Darboux sums and their applications	52
3. Lebesgue criterion and its applications	59
4. Mean theorem. Newton-Leibniz formula	63
5. The measure of a figure in \mathbb{R}^n	68
6. Areas of plane figures bounded by graphics of functions	75
7. The volume of a rotational solid	85
8. The length of a curve in \mathbb{R}^3	87
9. Approximate computation of definite integrals.	90
10. Problems and exercises	101
Chapter 3. Improper (generalized) integrals	105
1. More on limits of functions of one variable	105
2. Improper integrals of the first type	112
3. Improper integrals of the second type	122
4. Problems and exercises	135
Chapter 4. Integrals with parameters	137
1. Proper integrals with parameters	137
2. Improper integrals with parameters	147
3. Euler's functions gamma and beta	167
4. Problems and exercises	179
Chapter 5. Line integrals	181
1. The mass of a wire. Line integrals of the first type.	181
2. Line integrals of the second type.	194

3. Independence on path. Conservative fields	201
4. Computing plane areas with line integrals	211
5. Supplementary remarks on line integrals	216
6. Problems and exercises	221
Chapter 6. Double integrals	225
1. Double integrals on rectangles.	225
2. Double integrals on an arbitrary bounded domain	233
3. Green formula. Applications.	244
4. Change of variables in double integrals. Polar coordinates.	251
5. Problems and exercises	263
Chapter 7. Triple integrals	265
1. What is a triple integral on a parallelepiped?	265
2. Triple integrals on a general domain.	271
3. Iterative formulas for a general space domain	278
4. Change of variables in a triple integral	286
5. Problems and exercises	303
Chapter 8. Surface integrals	305
1. Deformation of a $2D$ -domain into a space surface	305
2. Surface integral of the first type. The mass of a $3D$ -lamina.	313
3. Flux of a vector field through an oriented surface.	319
4. Problems and exercises	335
Chapter 9. Gauss and Stokes formulas. Applications	339
1. Gauss formula (divergence theorem)	339
2. A mathematical model of the heat flow in a solid	348
3. Stokes Theorem	349
4. Problems and exercises	357
Chapter 10. Some remarks on complex functions integration	359
1. Complex functions integration	359
2. Applications of residues formula	377
3. Problems and exercises	385
Appendix A. Exam samples	387
1. June, 2010	387
2. June, 2010	387
3. June, 2010	387
4. June, 2010	388
5. September, 2010	388
6. September, 2010	388
Appendix B. Basic antiderivatives	389

CONTENTS

5

Appendix. Index

391

Appendix. Bibliography

397

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