Teachers Teaching with Technology

T^3 Scotland

T^3 EUROPE

Polynomials

POLYNOMIALS

Aim

To demonstrate how the TI-83 can be used to facilitate a fuller understanding of polynomials and show clearly the relationship between the algebraic solution and the graphical solution.

Objectives

Mathematical objectives

By the end of this unit you should know:

- how torecognise features of various polynomials
- the relationship between the graph of a sitution and the algebra used to describe it
- the relationship between roots and factors
- how to factorise and solve polynomial equations
- how to find approximate roots of a polynomial by decimal search
- how to find polynomial coefficients

Calculator objectives

By the end of this unit you should be able to

- draw graphs using [Y=]
- alter the display of a graph using [WINDOW] and [ZOOM].
- use the [2nd][TABLE] function with appropriate setting, using [2nd][TBLSET]

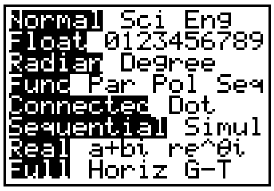
Factorising and Solving Polynomial Equations

Calculator skills sheet

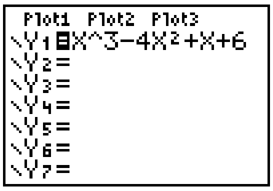
Using a TI - 83 to assist you in solving polynomials not only reduces the chances of you making a silly error but also makes the whole process much faster. It can also help you gain a fuller understanding of the mathematics.
Here is a typical question from a textbook and a method for solution.

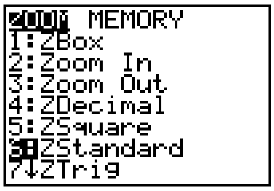
Find the roots of $x^3 - 4x^2 + x + 6 = 0$

Before going any further check that the [MODE] screen looks like this, particularly ensure that the TI is not rounding answers by highlighting Float.
Rounded answers could appear as non-existent roots.

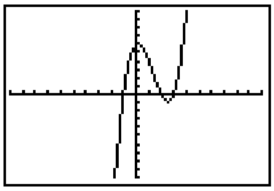


Enter the function on the [Y=] screen and graph the result on the [ZOOM 6:ZStandard] window range.
For some functions [ZOOM 4:ZDecimal] is appropriate.

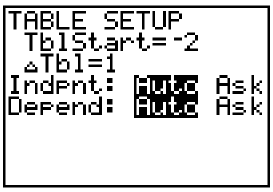


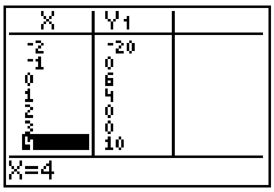


The graph of this function shows that it has roots at values which look like -1, 2 and 3.



This can be confirmed by looking at a table of values over this range. Using the [2nd][TBLSET] screen and then [2nd][TABLE] we can see that the values of the function are indeed zero at -1, 2 and 3.





Hence we can now say that $x = -1$, $x = 2$ and $x = 3$ are roots.

Once we have obtained the roots from the calculator it is easy to say what the factors of the equation are and so this same method can be used to fully factorise a polynomial.
The process to be used is almost identical.

Example 1

Find the roots of $x^3 - 4x^2 + x + 6 = 0$

WORKED SOLUTION

From graphic calculator root at $x = -1$

-1	1	-4	1	6
		-1	5	-6
	1	-5	6	**0**

No remainder $x = -1$ is a root and $(x + 1)$ is a factor. (Factor Theorem)

$x^2 - 5x + 6$

This quadratic must factorise to give $(x - 2)$ and $(x - 3)$. We already know this from the graph and table but can confirm it by multiplying out the bracket or by factorising the quadratic

$$x^3 - 4x^2 + x + 6 = (x+1)(x^2 - 5x + 6) = 0$$
$$= (x+1)(x-2)(x-3) = 0$$
$$\therefore x + 1 = 0 \text{ or } x - 2 = 0 \text{ or } x - 3 = 0$$
$$\therefore x = -1 \quad \text{or} \quad x = 2 \quad \text{or} \quad x = 3$$

Exercise 1

Solve these Polynomial Equations.

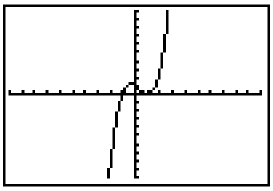
1. $x^3 - 6x^2 + 11x - 6 = 0$
2. $x^3 + 4x^2 + 5x + 2 = 0$
3. $x^3 - 3x^2 - 16x + 48 = 0$
4. $x^4 - 3x^2 - 2x = 0$
5. $x^4 - x^2 + 4x - 4 = 0$

Example 2

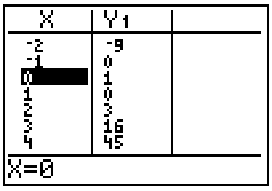
Fully factorise $x^3 - x^2 - x + 1$

This function is a cubic (polynomial of degree 3) which means it may have:

Roots	Factors
3 i.e $x = a$, $x = b$, $x = c$	3 Unique Linear Factors $(x - a)(x - b)(x - c)$
2 i.e $x = a$, $x = b$, $x = b$	1 Unique Linear Factor & 1 Repeated pair $(x - a)(x - b)(x - b)$
1 i.e $x = a$	1 Linear Factor & 1 Non-factorising Quadratic



[ZOOM 6:ZStandard]



X	Y1
-2	-9
-1	0
0	1
1	0
2	3
3	16
4	45

X=0

From the TI it can be seen that this function has 1 unique root and 1 pair of coincidental roots, i.e. it must have 1 unique linear factor and 1 pair of repeated linear factors. The table display shows that the roots are $x = -1$ and $x = 1$, from the graph we can see that the roots at $x = 1$, are coincidental.

The roots of this function are $x = -1$, $x = 1$ and $x = 1$.

The factors must be $(x + 1)$ and $(x - 1)$ and $(x - 1)$

WORKED SOLUTION

From graphic calculator root at $x = -1$

-1	1	-1	-1	1
		-1	2	-1
	1	-2	1	**0**

0 ← No remainder $x = -1$ is a root and $(x + 1)$ is a factor. (Factor Theorem)

$x^2 - 2x + 1$ ← This quadratic must factorise to give $(x - 1)$ and $(x - 1)$.

$$x^3 - x^2 - x + 1 = (x + 1)(x^2 - 2x + 1)$$
$$= (x + 1)(x - 1)(x - 1)$$