Understanding Basic Calculus

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Dedicated to all the people who have helped me in my life.

Preface

This book is a revised and expanded version of the lecture notes for *Basic Calculus* and other similar courses offered by the Department of Mathematics, University of Hong Kong, from the first semester of the academic year 1998-1999 through the second semester of 2006-2007. It can be used as a textbook or a reference book for an introductory course on one variable calculus.

In this book, much emphasis is put on explanations of concepts and solutions to examples. By reading the book carefully, students should be able to understand the concepts introduced and know how to answer questions with justification. At the end of each section (except the last few), there is an exercise. Students are advised to do as many questions as possible. Most of the exercises are simple drills. Such exercises may not help students understand the concepts; however, without practices, students may find it difficult to continue reading the subsequent sections.

Chapter 0 is written for students who have forgotten the materials that they have learnt for HKCEE Mathematics. Students who are familiar with the materials may skip this chapter.

Chapter 1 is on sets, real numbers and inequalities. Since the concept of sets is new to most students, detail explanations and elaborations are given. For the real number system, notations and terminologies that will be used in the rest of the book are introduced. For solving polynomial inequalities, the method will be used later when we consider where a function is increasing or decreasing as well as where a function is convex or concave. Students should note that there is a shortcut for solving inequalities, using the Intermediate Value Theorem discussed in Chapter 3.

Chapter 2 is on functions and graphs. Some materials are covered by HKCEE Mathematics. New concepts introduced include domain and range (which are fundamental concepts related to functions); composition of functions (which will be needed when we consider the Chain Rule for differentiation) and inverse functions (which will be needed when we consider exponential functions and logarithmic functions).

In Chapter 3, intuitive idea of limit is introduced. Limit is a fundamental concept in calculus. It is used when we consider differentiation (to define derivatives) and integration (to define definite integrals). There are many types of limits. Students should notice that their definitions are similar. To help students understand such similarities, a summary is given at the end of the section on two-sided limits. The section of continuous functions is rather conceptual. Students should understand the statements of the Intermediate Value Theorem (several versions) and the Extreme Value Theorem.

In Chapters 4 and 5, basic concepts and applications of differentiation are discussed. Students who know how to work on limits of functions at a point should be able to apply definition to find derivatives of "simple" functions. For more complicated ones (polynomial and rational functions), students are advised not to use definition; instead, they can use rules for differentiation. For application to curve sketching, related concepts like critical numbers, local extremizers, convex or concave functions etc. are introduced. There are many easily confused terminologies. Students should distinguish whether a concept or terminology is related to a function, to the x-coordinate of a point or to a point in the coordinate plane. For applied extremum problems, students