DIFFERENTIAL AND INTEGRAL CALCULUS, I

LECTURE NOTES (TEL AVIV UNIVERSITY, FALL 2009)

Contents

Date: 29 October, 2009.

Preliminaries

Preparatory reading. These books are intended for high-school students who like math. All three books are great, my personal favorite is the first one.

(1) R. Courant, H. Robbins, I. Stewart, What is mathematics, Oxford, 1996 (or earlier editions).
(2) T. W. Korner, The pleasures of counting, Cambridge U. Press, 1996.
(3) K. M. Ball, Strange curves, counting rabbits, and other mathematical explorations, Princeton University Press, 2003.

Reading. There are many good textbooks in analysis, though I am not going to follow any of them too closely. The following list reflects my personal taste:

(1) V. A. Zorich, Mathematical analysis, vol.1, Springer, 2004.
(2) A. Browder, Mathematical analysis. An introduction. Undergraduate Texts in Mathematics. Springer-Verlag, New York, 1996.
(3) R. Courant and F. John, Introduction to calculus and analysis, vol.1, Springer, 1989 (or earlier editions).
(4) D. Maizler, Infinitesimal calculus (in Hebrew).
(5) G. M. Fihtengol'tz, Course of Differential and Integral Calculus, vol. I (in Russian)
(6) E. Hairer, G. Wanner, Analysis by its history, Springer, 1996.

The last book gives a very interesting and motivated exposition of the main ideas of this course given in the historical perspective.

You may find helpful informal discussions of various ideas related to this course (as well to the other undergraduate courses) at the web page of Timothy Gowers:

`www.dpmms.cam.ac.uk/~wtg10/mathsindex.html`

I suppose that the students attend in parallel with this course the course "Introduction to the set theory", or the course "Discrete Mathematics". The notes (in Hebrew) of Moshe Jarden might be useful:

`www.math.tau.ac.il/~jarden/Courses/set.pdf`

Problem books. For those of you who are interested to try to solve more difficult and interesting problems and exercises, I strongly recommend to look at two excellent collections of problems:

(1) B. M. Makarov, M. G. Goluzina, A. A. Lodkin, A. N. Podkorytov, Selected problems in real analysis, American Mathematical Society, 1992.
(2) G. Polya, G. Szegö, Problems and theorems in analysis (2 volumes) Springer, 1972 (there are earlier editions).

Basic notation.

Symbols from logic.

$\vee$ or
$\wedge$ and
$\neg$ negation
$\Longrightarrow$ yields
$\Longleftrightarrow$ is equivalent to

Example: $(x^2 - 3x + 2 = 0) \Longleftrightarrow ((x = 1) \vee (x = 2))$

Quantifiers:

$\exists$ exists
$\exists!$ exists and unique (warning: this notation isn't standard)
$\forall$ for every

Set-theoretic notation.

$\in$ belongs
$\notin$ does not belong
$\subset$ subset
$\emptyset$ empty set
$\cap$ intersection of sets
$\cup$ union of sets
$\#(X)$ cardinality of the set X
$X \setminus Y = \{x \in X : x \notin Y\}$ complement to Y in X

Example: $(X \subset Y) := \forall x \ ((x \in X) \Longrightarrow (x \in Y))$

We shall freely operate with these notion during the course. Usually, the sets we deal with are subsets of the set of real numbers $\mathbb{R}$.

Subsets of reals:

$\mathbb{N}$ natural numbers (positive integers)
$\mathbb{Z}$ integers
$\mathbb{Z}_+ = \mathbb{N} \bigcup \{0\}$ non-negative integers
$\mathbb{Q}$ rational numbers
$\mathbb{R}$ real numbers
$[a, b] := \{x \in \mathbb{R} : a \le x \le b\}$ closed interval (one point sets are closed intervals as well)
$(a, b) := \{x \in \mathbb{R} : a < x < b\}$ open interval
$(a, b]$ and $[a, b)$ semi-open intervals

Sums and products.

$$\sum_{j=1}^{n} a_j = a_1 + a_2 + ... + a_n$$

$$\prod_{j=1}^{n} a_j = a_1 \cdot a_2 \cdot ... \cdot a_n$$