THE CALCULUS INTEGRAL

Brian S. Thomson
Simon Fraser University

ClassicalRealAnalysis.com

This text is intended as an outline for a rigorous course introducing the basic elements of integration theory to honors calculus students or for an undergraduate course in elementary real analysis. Since "all" exercises are worked through in the appendix, the text is particularly well suited to self-study.

For further information on this title and others in the series visit our website.

www.classicalrealanalysis.com

There are PDF files of all of our texts available for download as well as instructions on how to order trade paperback copies. We always allow access to the full content of our books on GOOGLE BOOKS and on the AMAZON *Search Inside the Book* feature.

Cover Image: Sir Isaac Newton

And from my pillow, looking forth by light
Of moon or favouring stars, I could behold
The antechapel where the statue stood
Of Newton with his prism and silent face,
The marble index of a mind for ever
Voyaging through strange seas of Thought, alone.

. . . William Wordsworth, *The Prelude*.

Citation: *The Calculus Integral*, Brian S. Thomson, ClassicalRealAnalysis.com (2010), [ISBN 1442180951]

Date PDF file compiled: June 19, 2011

BETA VERSION β 1.0

The file or paperback that you are reading should be considered a work in progress. In a classroom setting make sure all participants are using the same beta version. We will add and amend, depending on feedback from our users, until the text appears to be in a stable condition.

ISBN: 1442180951

EAN-13: 9781442180956

CLASSICALREALANALYSIS.COM

PREFACE

There are plenty of calculus books available, many free or at least cheap, that discuss integrals. Why add another one?

Our purpose is to present integration theory at an honors calculus level and in an easier manner by defining the definite integral in a very traditional way, but a way that avoids the equally traditional Riemann sums definition.

Riemann sums enter the picture, to be sure, but the integral is defined in the way that Newton himself would surely endorse. Thus the fundamental theorem of the calculus starts off as the definition and the relation with Riemann sums becomes a theorem (not the definition of the definite integral as has, most unfortunately, been the case for many years).

As usual in mathematical presentations we all end up in the same place. It is just that we have taken a different route to get there. It is only a pedagogical issue of which route offers the clearest perspective. The common route of starting with the definition of the Riemann integral, providing the then necessary detour into improper integrals, and ultimately heading towards the Lebesgue integral is arguably not the best path although it has at least the merit of historical fidelity.

Acknowledgments

I have used without comment material that has appeared in the textbook

> [TBB] *Elementary Real Analysis*, 2nd Edition, B. S. Thomson, J. B. Bruckner, A. M. Bruckner, ClassicalRealAnalyis.com (2008).

I wish to express my thanks to my co-authors for permission to recycle that material into the idiosyncratic form that appears here and their encouragement (or at least lack of discouragement) in this project.

I would also like to thank the following individuals who have offered feedback on the material, or who have supplied interesting exercises or solutions to our exercises: *[your name here], . . .*

Note to the instructor

Since it is possible that some brave mathematicians will undertake to present integration theory to undergraduates students using the presentation in this text, it would be appropriate for us to address some comments to them.

What should I teach the weak calculus students?

Let me dispense with this question first. Don't teach them this material, which is aimed much more at the level of an honor's calculus course. I also wouldn't teach them the Riemann integral. I think a reasonable outline for these students would be this:

1. An informal account of the indefinite integral formula
$$\int F'(x)\,dx = F(x)+C$$
just as an antiderivative notation with a justification provided by the mean-value theorem.

2. An account of what it means for a function to be continuous on an interval $[a,b]$.

3. The definition
$$\int_a^b F'(x)\,dx = F(b)-F(a)$$
for continuous functions $F : [a,b] \to \mathbb{R}$ that are differentiable at all[1] points in (a,b). The mean-value theorem again justifies the definition. You won't need improper integrals, e.g.,
$$\int_0^1 \frac{1}{\sqrt{x}}\,dx = \int_0^1 \frac{d}{dx}\left(2\sqrt{x}\right)\,dx = 2-0.$$

4. Any properties of integrals that are direct translations of derivative properties.

5. The Riemann sums *identity*
$$\int_a^b f(x)\,dx = \sum_{i=1}^n f(\xi_i^*)(x_i - x_{i-1})$$
where the points ξ_i^* that make this precise are selected by the mean-value theorem.

[1]...or all but finitely many points