

A Course  
on  
Convex Geometry

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revised version 2009/2010

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# Preface

The following notes were written before and during the course on *Convex Geometry* which was held at the University of Karlsruhe in the winter term 2002/2003. Although this was the first course on this topic which was given in English, the material presented was based on previous courses in German which have been given several times, mostly in summer terms. In comparison with these previous courses, the standard program was complemented by sections on surface area measures and projection functions as well as by a short chapter on integral geometric formulas. The idea here was to lay the basis for later courses on *Stochastic Geometry*, *Integral Geometry* etc., which usually follow in a subsequent term.

The exercises at the end of each section contain all the weekly problems which were handed out during the course and discussed in the weekly exercise session. Moreover, I have included a few additional exercises (some of which are more difficult) and even some hard or even unsolved problems. The list of exercises and problems is far from being complete, in fact the number decreases in the later sections due to the lack of time while preparing these notes.

I thank Matthias Heveling and Markus Kiderlen for reading the manuscript and giving hints for corrections and improvements.

Karlsruhe, February 2003

Wolfgang Weil

During repetitions of the course in 2003/2004 and 2005/2006 a number of misprints and small errors have been detected. They are corrected in the current version. Also, additional material and further exercises have been added.

Karlsruhe, October 2007

Wolfgang Weil

During the courses in 2008/2009 (by D. Hug) and 2009/2010 (by W. Weil) these lecture notes have been revised and extended again. Also, some pictures have been included.

Karlsruhe, October 2009

Daniel Hug and Wolfgang Weil



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