Linear Algebra

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Preface

This "book" grew out of a series of twenty five lecture notes for a sophomore linear algebra class taught at the University of California, Davis. The audience was primarily engineering students and students of pure sciences, some of whom may go on to major in mathematics. It was motivated by the lack of a book that taught students basic structures of linear algebra without overdoing mathematical rigor or becoming a mindless exercise in crunching recipes at the cost of fundamental understanding. In particular we wanted a book that was suitable for all students, not just math majors, that focussed on concepts and developing the ability to think in terms of abstract structures in order to address the dizzying array of seemingly disparate applications that can all actually be addressed with linear algebra methods.

In addition we had practical concerns. We wanted to offer students a online version of the book for free, both because we felt it our academic duty to do so, but also because we could seamlessly link an online book to a myriad of other resources–in particular WeBWorK exercises and videos. We also wanted to make the LaTeX source available to other instructors so they could easily customize the material to fit their own needs. Finally, we wanted to restructure the way the course was taught, by getting the students to direct most of their effort at more difficult problems where they had to think through concepts, present well-thought out logical arguments and learn to turn word problems into ones where the usual array of linear algebra recipes could take over.

How to Use the Book

At the end of each chapter there is a set of review questions. Our students found these very difficult, mostly because they did not know where to begin, rather than needing a clever trick. We designed them this way to ensure that students grappled with basic concepts. Our main aim was for students to master these problems, so that we could ask similar high caliber problems on midterm and final examinations. This meant that we did have to direct resources to grading some of these problems. For this we used two tricks. First we asked students to hand in more problems than we could grade, and then secretly selected a subset for grading. Second, because there are more review questions than what an individual student could handle, we split the class into groups of three or four and assigned the remaining problems to them for grading. Teamwork is a skill our students will need in the workplace; also it really enhanced their enjoyment of mathematics.

Learning math is like learning to play a violin–many "technical exercises" are necessary before you can really make music! Therefore, each chapter has a set of dedicated WeBWorK "skills problems" where students can test that they have mastered basic linear algebra skills. The beauty of WeBWorK is that students get instant feedback and problems can be randomized, which means that although students are working on the same types of problem, they cannot simply tell each other the answer. Instead, we encourage them to explain to one another how to do the WeBWorK exercises. Our experience is that this way, students can mostly figure out how to do the WeBWorK problems among themselves, freeing up discussion groups and office hours for weightier issues. Finally, we really wanted our students to carefully read the book. Therefore, each chapter has several very simple WeBWorK "reading problems". These appear as links at strategic places. They are very simple problems that can answered rapidly if a student has read the preceding text.

The Material

We believe the entire book can be taught in twenty five fifty minute lectures to a sophomore audience that has been exposed to a one year calculus course. Vector calculus is useful, but not necessary preparation for this book, which attempts to be self-contained. Key concepts are presented multiple times, throughout the book, often first in a more intuitive setting, and then again in a definition, theorem, proof style later on. We do not aim for students to become agile mathematical proof writers, but we do expect them to be able to show and explain why key results hold. We also often use the review exercises to let students discover key results for themselves; before they are presented again in detail later in the book.

Linear algebra courses run the risk of becoming a conglomeration of learn-by-rote recipes involving arrays filled with numbers. In the modern computer era, understanding these recipes, why they work, and what they are for is more important than ever. Therefore, we believe it is crucial to change the students' approach to mathematics right from the beginning of the course. Instead of them asking us "what do I do here?", we want them to ask "why would I do that?" This means that students need to start to think in terms of abstract structures. In particular, they need to rapidly become conversant in sets and functions–the first WeBWorK set will help them brush up these skills.

There is no best order to teach a linear algebra course. The book has been written such that instructors can reorder the chapters (using the LaTeX source) in any (reasonable) order and still have a consistent text. We hammer the notions of abstract vectors and linear transformations hard and early, while at the same time giving students the basic matrix skills necessary to perform computations. Gaussian elimination is followed directly by an "exploration chapter" on the simplex algorithm to open students minds to problems beyond standard linear systems ones. Vectors in $\mathbb{R}^n$ and general vector spaces are presented back to back so that students are not stranded with the idea that vectors are just ordered lists of numbers. To this end, we also labor the notion of all functions from a set to the real numbers. In the same vein linear transformations and matrices are presented hand in hand. Once students see that a linear map is specified by its action on a limited set of inputs, they can already understand what a basis is. All the while students are studying linear systems and their solution sets, so after matrices determinants are introduced. This material can proceed rapidly since elementary matrices were already introduced with Gaussian elimination. Only then is a careful discussion of spans, linear independence and dimension given to ready students for a thorough treatment of eigenvectors and diagonalization. The dimension formula therefore appears quite late, since we prefer not to elevate rote computations of column and row spaces to a pedestal. The book ends with applications–least squares and singular values. These are a fun way to end any lecture course. It would also be quite easy to spend any extra time on systems of differential equations and simple Fourier transform problems.