AMSI
AUSTRALIAN MATHEMATICAL SCIENCES INSTITUTE

INTERNATIONAL CENTRE OF EXCELLENCE FOR EDUCATION IN MATHEMATICS

The Improving Mathematics Education in Schools (TIMES) Project

TRIGONOMETRIC FUNCTIONS

A guide for teachers - Year 10

MEASUREMENT AND GEOMETRY : Module 25

June 2011

YEAR 10

Trigonometric Functions

(Measurement and Geometry: Module 25)

For teachers of Primary and Secondary Mathematics

510

Cover design, Layout design and Typesetting by Claire Ho

The Improving Mathematics Education in Schools (TIMES) Project 2009-2011 was funded by the Australian Government Department of Education, Employment and Workplace Relations.

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YEAR 10

THE TRIGONOMETRY FUNCTIONS

ASSUMED KNOWLEDGE

- Familiarity with the material in the modules, *Introduction to Trigonometry* and *Further Trigonometry*.
- Knowledge of basic coordinate geometry.
- Introductory graphs and functions.
- Facility with simple algebra, formulas and equations.

MOTIVATION

In the module, *Further Trigonometry*, we saw how to use points on the unit circle to extend the definition of the trigonometric ratios to include obtuse angles. That same construction can be extended to angles between 180° and 360° and beyond. The sine, cosine and tangent of negative angles can be defined as well.

Once we can find the sine, cosine and tangent of any angle, we can use a table of values to plot the graphs of the functions $y = \sin x$, $y = \cos x$ and $y = \tan x$. In this module, we will deal only with the graphs of the first two functions.

The graphs of the sine and cosine functions are used to model wave motion and form the basis for applications ranging from tidal movement to signal processing which is fundamental in modern telecommunications and radio-astronomy. This provides a breathtaking example of how a simple idea involving geometry and ratio was abstracted and developed into a remarkably powerful tool that has changed the world.

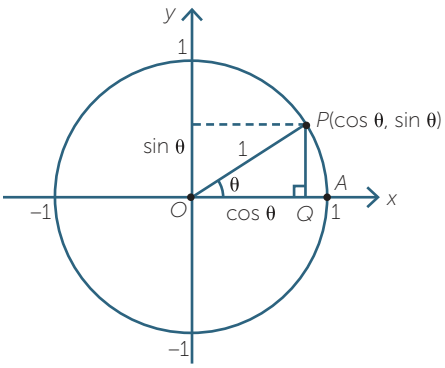
CONTENT

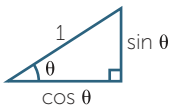
ANGLES IN THE FOUR QUADRANTS

Redefining the Trigonometric Ratios

We begin by taking the circle of radius 1, centre the origin, in the plane. From the point P on the circle in the first quadrant we can construct a right-angled triangle POQ with O at the origin and Q on the x-axis.

We mark the angle POQ as θ.





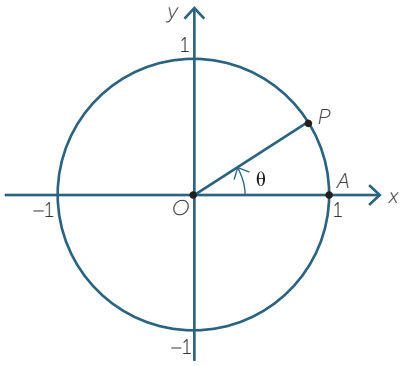
Since the length $OQ = \cos\theta$ is the x-coordinate of P, and $PQ = \sin\theta$ is the y-coordinate of P, we see that the point P has coordinates

$$(\cos\theta, \sin\theta).$$

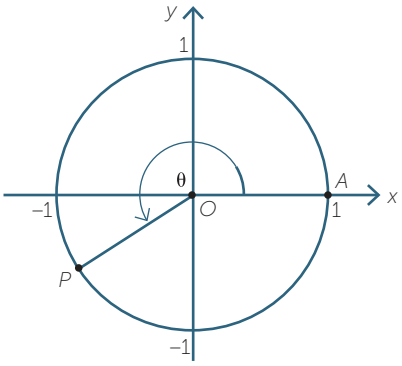
We measure angles anticlockwise from OA and call these **positive angles.** Angles measured clockwise from OA are called **negative angles**. For the time being we will concentrate on positive angles between 0° and 360°.

Since each angle θ determines a point P on the unit circle, we will define

- the cosine of θ to be the x-coordinate of the point P
- the sine of θ to be the y-coordinate of the point P.



$0° < \theta < 90°$



$180° < \theta < 270°$