Mathematics Learning Centre

The University of Sydney

# Introduction to Trigonometric Functions

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**Acknowledgements**

A significant part of this manuscript has previously appeared in a version of this booklet published in 1986 by Peggy Adamson. In rewriting this booklet, I have relied a great deal on Peggy's ideas and approach for Chapters 1, 2, 3, 4, 5 and 7. Chapter 6 appears in a similar form in the booklet, Introduction to Differential Calculus, which was written by Christopher Thomas.

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# Contents

# 1 Introduction

You have probably met the trigonometric ratios cosine, sine, and tangent in a right angled triangle, and have used them to calculate the sides and angles of those triangles.

In this booklet we review the definition of these trigonometric ratios and extend the concept of cosine, sine and tangent. We define the cosine, sine and tangent as functions of all real numbers. These trigonometric functions are extremely important in science, engineering and mathematics, and some familiarity with them will be assumed in most first year university mathematics courses.

In Chapter 2 we represent an angle as radian measure and convert degrees to radians and radians to degrees. In Chapter 3 we review the definition of the trigonometric ratios in a right angled triangle. In Chapter 4, we extend these ideas and define cosine, sine and tangent as functions of real numbers. In Chapter 5, we discuss the properties of their graphs. Chapter 6 looks at derivatives of these functions and assumes that you have studied calculus before. If you haven't done so, then skip Chapter 6 for now. You may find the Mathematics Learning Centre booklet: *Introduction to Differential Calculus* useful if you need to study calculus. Chapter 7 gives a brief look at inverse trigonometric functions.

## 1.1 How to use this booklet

You will not gain much by just reading this booklet. Mathematics is not a spectator sport! Rather, have pen and paper ready and try to work through the examples before reading their solutions. Do **all** the exercises. It is important that you try hard to complete the exercises, rather than refer to the solutions as soon as you are stuck.

## 1.2 Objectives

By the time you have completed this booklet you should:

- know what a radian is and know how to convert degrees to radians and radians to degrees;
- know how cos, sin and tan can be defined as ratios of the sides of a right angled triangle;
- know how to find the cos, sin and tan of $\frac{\pi}{6}$, $\frac{\pi}{4}$ and $\frac{\pi}{2}$;
- know how cos, sin and tan functions are defined for all real numbers;
- be able to sketch the graph of certain trigonometric functions;
- know how to differentiate the cos, sin and tan functions;
- understand the definition of the inverse function $f^{-1}(x) = \cos^{-1}(x)$.